

NEWNAN JONES WHITTAKER ESCHENBACH LAVELLE

#### Fourth Canadian Edition

# ENGINEERING ECONOMIC ANALYSIS

NEWNAN
JONES
WHITTAKER
ESCHENBACH
LAVELLE





Oxford University Press is a department of the University of Oxford. It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide. Oxford is a registered trade mark of Oxford University Press in the UK and in certain other countries.

Published in Canada by
Oxford University Press
8 Sampson Mews, Suite 204,
Don Mills, Ontario M3C 0H5 Canada

www.oupcanada.com

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Database right Oxford University Press (maker)

First Canadian Edition published in 2006 Second Canadian Edition published in 2010 Third Canadian Edition published in 2014

Original edition published by Oxford University Press, Inc., 198 Madison Avenue, New York, N.Y. 10016-4314, USA. Copyright © 2012, 2009, 2004 by Oxford University Press, Inc.

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#### Library and Archives Canada Cataloguing in Publication

Newnan, Donald G., author

Engineering economic analysis / Donald G. Newnan, John Jones, John Whittaker, Ted G. Eschenbach, Jerome P. Lavelle. — Fourth Canadian edition.

Includes bibliographical references and index.
Issued in print and electronic formats.
ISBN 978-0-19-902511-4 (hardcover).—ISBN 978-0-19-902522-0 (PDF)

1. Engineering economy—Textbooks. 2. Textbooks. I. Jones, John (John Dewey), author II. Whittaker, J. D., author III. Eschenbach, Ted, author IV. Lavelle, Jerome P., author V. Title.

TA177.4.N486 2018

658.15

C2017-905532-1

C2017-905533-X

Cover image: © Ravil Sayfullin/123RF Cover and interior design: Sherill Chapman

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Printed and bound in Canada

1 2 3 4 — 21 20 19 18

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# Preface and Acknowledgements

Engineering is the application of knowledge to develop practical and economical solutions to the problems that confront our civilization. Canada, with an area of 9,984,670 km², is the second-largest country in the world. But with a population of about 35,300,000, Canada has less than 0.5% of the world's people. In population density we rank about 230th, with 3.9 people per square kilometre of land area. Because much of the country is a cold and unforgiving place, Canadians have needed all the engineering skills they could muster in order to exist in this formidable environment and, in doing so, to consistently achieve a position in the top five on the United Nations list of the best countries to live in.

The history of this country is a history of engineering innovation and adaptability. The ingenious design of the birchbark canoe enabled First Nations people to travel thousands of kilometres and develop trading routes that spanned the country. In the 1800s, engineers from Britain taught us how to construct canals to circumvent rapids; we built on that knowledge, and with the opening of the St Lawrence Seaway in 1959, ocean-going ships of commerce were able to bypass even so great an obstacle as Niagara Falls and penetrate the heart of the continent.

The promise of a transcontinental railway was what brought British Columbia into the Canadian Confederation, and again the engineers responded, completing the project in 1885 and opening the western provinces to settlement and development.

Canoes work well in the summer, but winter travel poses special challenges. Joseph-Armand Bombardier attached a Ford engine to a sleigh, which, with continuing engineering refinements, evolved into their snowmobile, the Ski-doo, and its inventor's company became the aircraft and light-rail-transit manufacturer Bombardier, Inc.

The need to communicate with northern settlements led Canadian engineers to develop the Anik communications satellite. And our need to communicate, wherever we happen to be, led a University of Waterloo engineering student, Mike Lazaridis, to develop the BlackBerry and create the company Research In Motion (RIM).

Engineering has always been a mixture of need, practicality, and economics. The concepts of ethics, sustainability, and environmental stewardship are now being added to that mix. This book introduces the economic decisions that accompany engineering design, but it also includes questions that explore those other aspects. Each chapter begins with a vignette describing a Canadian engineering challenge and inviting the reader to think about the broader implications of engineering decisions.

# **Changes to the Fourth Edition**

This edition has significant improvements in organization and coverage. Before going chapter by chapter, we'd like to point out a few global changes made to keep this text the most current and most useful in today's courses:

- A new discussion on the environmental impacts of engineering projects has been added.
- Additional Canadian examples are included throughout.
- All units are now in SI.
- We have expanded coverage of money, inflation, and the Bank of Canada.
- The majority of in-chapter Excel explanations have been removed and placed online.

Numerous improvements to the content and wording have been made throughout the text. The text has been condensed and some print material, such as the compound interest tables, has been placed online.

Changes from the third edition include the following:

- Chapters 1 and 2 have been combined and condensed into one chapter.
- Chapter 8 has been eliminated, and some material has been distributed in other chapters (information on incremental analysis to Chapter 7, future worth content to Chapter 5, benefit-cost ratio to Chapter 8, sensitivity and break-even content to Chapter 10).
- Chapter 17 has been moved up to become Chapter 2.
- Chapter 16 has been moved up to become Chapter 8.
- A new chapter on project management has been added.
- In Chapter 12, the explanation for calculation of taxes for individuals has been revised for clarity, and the calculation of after-tax cash flows has been revised.

#### **Icons**



These marginal icons appear throughout the text to highlight important key concepts.



These icons appear in the Problems section at the end of each chapter to signify that the answer to the problem can be found on the companion website: www.oupcanada.com/Newnan4Ce

# A Special Word on Spreadsheets

In this edition, the authors have deliberately omitted any discussion of how to use any form of spreadsheet software. Since there are many spreadsheet programs available, and since their details change frequently, these details are better obtained from online documentation. Students will notice rather quickly that the use of a suitable spreadsheet program can greatly reduce the labour involved in solving economic problems, and they should be encouraged to do this.

# Acknowledgements

This textbook is the result of the hard work of a lot of people for over 35 years. I, along with Oxford University Press Canada, would like to gratefully acknowledge the following reviewers, as well as those who wished to remain anonymous:

Soha Eid Moussa, University of Guelph Tiffany Bayley, University of Waterloo Scott Flemming, Dalhousie University Michael Bennett, UOIT Juan Pernia, Lakehead University Michael H. Wang, University of Windsor Nadine Ibrahim, University of Toronto Their thoughtful comments and suggestions have helped to shape the fourth edition of this text.

For the fourth Canadian edition, I would like to thank all the people at OUP Canada—including Meg Patterson, Lauren Wing, Suzanne Clark, and Liana Spada—and Doug Linzey of Agricola Communications. Finally, I want to thank my wife, Joan, and my daughter and editorial critic Ali.

John Jones Simon Fraser University, 2017

# Economic Decisions, Engineering Costs, and Cost Estimating

#### **Alternative Futures**

There is no better place to start an engineering economics text than with a discussion of the future of the car, for that question embodies many of the technology and economic factors involved.

Hybrids, plug-in electrics, turbocharged diesels, and advanced gasoline engines all compete for the buyer's (and driver's) attention. In 2016 the lineup included the following sedans:

Model	Technology	Litres per 100 km	MSRP
Nissan Leaf	Electric	2.4	\$32,700
Chevrolet Volt	Electric	3.8	\$40,000
Toyota Prius	Hybrid	4.7	\$26,000
Honda Civic	Hybrid	5.8	\$16,000
BMW X5 xDrive35d	Diesel	6.2	\$87,000
Chevrolet Cruz	Diesel	5.1	\$24,670
Chevrolet Cruze Eco	Gasoline	5.6	\$16,175
Toyota Yaris	Gasoline	4.6	\$16,365



Choosing the right vehicle is a complex problem, and evaluating the choices requires much more information than the price and fuel mileage. Will you be driving in the country or city, or a mix? How far do you drive in a year? What will the future price of energy (gas, diesel, electricity) be? How long will you keep the car before you sell it? What will the resale value be? What financing terms do the sellers offer?

Electric vehicle technology has been slowly improving. Since 2012, the University of Waterloo has been challenging teams of high-school students to design and build all-electric vehicles to compete in a 12-volt and a 24-volt endurance race. In the first year of the competition, Bluevale Secondary School won the 12-volt and 24-volt races, with 41 and 61 laps, respectively. In 2013, they were narrowly edged out of first place in the 12-volt race by Resurrection Catholic Secondary School, who completed 51.9 laps, but retained first place in the 24-volt race with an impressive 95 laps.

But the gasoline and diesel engineers have not been standing still. Future drivers will continue to be offered an array of technologies and will have to use engineering economic analysis to find the correct choices.

#### **QUESTIONS TO CONSIDER**

- 1. What marketplace dynamics stimulate or suppress the development of alternative-fuel vehicles? What role, if any, does government have in these dynamics? What additional responsibilities should government have?
- 2. Develop a list of concerns and questions that consumers might have about the conversion to alternative-fuel vehicles. Which are economic factors, and which are non-economic?
- 3. Some environmentalists have proposed producing bio-fuels from crops as an alternative to fossil fuels. What ethical questions might this involve?
- 4. Power companies have a lot of extra capacity at night, and so there is a possibility of their offering off-peak power at a reduced cost. This could be used by people with plug-in hybrids. How much investment would a power company have to make in smart circuits for a time-variable pricing policy to be possible?

#### **LEARNING OBJECTIVES**

This chapter will help you

- describe the economic decision-making process
- choose appropriate economic criteria for different problems
- describe common ethical issues in engineering economic decisions
- define various cost concepts
- provide specific examples of how and why these engineering cost concepts are important
- define engineering cost estimating
- explain the three types of engineering estimates, as well as common difficulties encountered in making them
- use several common mathematical estimating models in cost estimating
- discuss the effect of the learning curve on cost estimates
- draw cash flow diagrams to show project costs and benefits

#### **KEY TERMS**

average cost book cost cash cost cash flow diagram estimating by analogy fixed cost forgone opportunity incremental cost learning curve learning curve percentage learning curve rate life-cycle costing marginal cost model building non-recurring cost opportunity cost out-of-pocket cost overhead

per unit model
power-sizing model
recurring cost
segmenting model
shadow price
sunk cost
triangulation
variable cost

#### A Sea of Problems

This book is about making decisions. It develops the tools for analyzing and solving economic problems commonly faced by engineers.

Our emphasis is on solving problems that confront firms in the marketplace, but many examples are problems students face in daily life. Let us start by looking at some of these problems.

#### **Simple Problems**

Many problems are simple, and good solutions do not require much time or effort.

- Should I pay by cash or credit card?
- Do I buy a semester parking pass or use the parking meters?
- Should we replace a burned-out motor?
- If we use three crates of an item a week, how many crates should we buy at a time?

#### **Intermediate Problems**

At the middle level of complexity, we find problems that are primarily economic.

- Should I buy or lease my next car?
- Which pieces of equipment should be chosen for a new assembly line?
- Should our office upgrade to the newest version of our project management software?
- Should I buy a one- or two-semester parking pass?
- Should we buy a low-cost press requiring three operators, or a more expensive one requiring only two operators?

#### **Complex Problems**

Complex problems represent a mixture of economic, political, and social and ethical elements.

• The question of building pipelines to export Albertan oil overseas is an example of a complex problem. Pipelines such as the proposed Northern Gateway from Alberta to Kitimat, BC, will benefit the Albertan economy and create jobs in the Albertan oil industry. On the other hand, some environmentalists believe that the risk of oil spills will endanger First Nations lands and the BC coastline. Globally,

- burning Alberta's oil will increase global warming, with possible adverse consequences worldwide.
- The choice of a girlfriend or a boyfriend (who may later become a spouse) is obviously complex. Economic analysis can be of little or no help.
- The annual budget of a corporation is an allocation of resources, and all projects are evaluated economically. The budget process is also heavily influenced by non-economic forces such as power struggles, geographical balancing, and effects on individuals, programs, and profits.

# The Role of Engineering Economic Analysis

Engineering economic analysis is most suitable for intermediate problems and the economic aspects of complex problems. Such problems have the following characteristics:

- 1. The problem is *important enough* to justify serious thought and effort.
- 2. The problem can't be worked out in one's head—that is, a careful analysis *requires that we organize* the problem and all the various consequences.
- 3. The problem has *economic aspects* that are important in reaching a decision.

Vast numbers of problems having these three characteristics are encountered in the business world and in people's personal lives, so engineering economic analysis is often required.

#### **Examples of Engineering Economic Analysis**

Engineering economic analysis focuses on costs, revenues, and benefits that occur at *different times*. For example, when a nuclear engineer designs a reactor, the construction costs occur in the near future; benefits to the users begin only when construction is finished, but continue for a long time.

Engineering economic analysis is used to answer many different questions.

- Which engineering projects are worthwhile? Has the mining or petroleum engineer shown that the mineral or oil deposit is worth developing?
- Which engineering projects should have a higher priority? Has the electrical engineer shown that replacing the plant's step-down transformer is more urgent than upgrading the power supply to the machine shop?
- How should the engineering project be designed? Has the mechanical engineer chosen the most economical size of motor? Has the software engineer calculated the development time for writing the next release of the software? Has the aeronautical engineer made the best trade-offs between (1) lighter materials that are expensive to buy but result in a plane that is cheaper to fly and (2) heavier materials that are cheap to buy but result in a plane that is more expensive to fly?

Engineering economic analysis can also be used to answer questions that are personally important.

How to achieve long-term financial goals: How much should you save each month
to buy a house, retire, or fund a trip around the world? Is going to graduate school
a good investment?

- How to compare different ways to finance purchases: Is it better to finance your car purchase by using the dealer's low-interest loan or by taking the rebate and borrowing money from your bank or credit union?
- *How to make short- and long-term investment decisions*: Should you buy a one- or two-semester parking pass? Is a higher salary better than stock options?

# **The Decision-Making Process**

Decision making requires that there be at least two alternatives. If only one course of action is available, there is no decision to make. Does decision making, then, consist of choosing from among alternative courses of action? Consider the following situation:

At a race track, a bettor was uncertain which horse to bet on in the next race. He closed his eyes and pointed his finger at the list of horses printed in the racing program. Upon opening his eyes, he saw that he was pointing to horse number 4. He hurried off to place his bet on that horse.

Does the racehorse selection represent a process of decision making? Clearly, it was a process of choosing among alternatives. But this process seems inadequate and irrational. We want to deal with *rational* decision making.

#### **Rational Decision Making**

Rational decision making can be a complex process. One possible systematic approach to this process is shown in Figure 1-1. Although the steps are shown sequentially, it is common for a decision maker to repeat steps, take them out of order, and do some steps simultaneously. For example, when a new alternative is identified, more data will be required. Or when the outcomes are summarized, it may become clear that the problem needs to be redefined or new goals established. The following sections describe the elements listed in Figure 1-1.

#### 1. Recognize the Problem

The starting point in rational decision making is recognizing that a problem exists.

Some years ago, for example, it was discovered that several species of ocean fish contained substantial concentrations of mercury. The decision-making process began with this discovery, and the rush was on to determine what should be done. Research revealed that fish taken from the ocean decades before and preserved in laboratories contained similar concentrations of mercury. Thus the problem had existed for a long time but had not been recognized.

In many situations, recognition is obvious and immediate. A car accident, a cheque that bounces, a burned-out motor, an exhausted supply of parts all produce the recognition of a problem. Once we are aware of the problem, we solve it as best we can.

#### 2. Define the Goal or Objective

The goal can be a grand, overall goal of a person or a firm. For example, a personal goal could be to lead a pleasant and meaningful life, whereas a firm's goal is usually to operate profitably.

But an objective need not be the grand, overall goal of a business or an individual. It may be narrow and specific: "I want to pay off the loan on my car by May" and "The plant must produce 300 golf carts in the next two weeks" are more limited objectives.

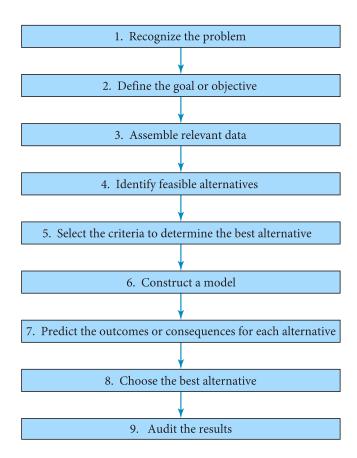


FIGURE 1-1 One possible flow chart of the decision-making process.

#### 3. Assemble Relevant Data

To make a good decision, you must assemble good information. In addition to all the published information, a vast quantity of information is not written down anywhere but is stored as individuals' knowledge and experience. There is also information that remains ungathered. A question like "How many people in your town would be interested in buying a pair of left-handed scissors?" cannot be answered by examining published data or by asking any one person. Market research or other data gathering would be required to obtain the desired information.

One part of the data that must be assembled is the time horizon of the problem. How long will the building or equipment last? How long will it be needed? Will it be scrapped, sold, or shifted to another use? In some cases, such as a road or a tunnel, the life may be centuries with regular maintenance and occasional rebuilding. However, a shorter period, such as 50 years, may be chosen as the time horizon so that decisions can be based on more reliable data.

In engineering decisions, an important source of data is a firm's own accounting system. These data must be examined carefully. That is because accounting data focus on past information, whereas engineering judgment must often be applied to estimate current and future values. For example, accounting records can show the past cost of buying computers, but engineering judgment is required to estimate their future cost.

Financial and cost accounting are designed to show the flow of money—specifically costs and benefits—in a company's operations. When costs are directly related to specific operations, there is no difficulty; but there are other costs unrelated to specific operations. These indirect costs, or **overhead**, are usually allocated to a company's operations and

products by some semi-arbitrary method. The results are generally satisfactory for cost-accounting purposes but may be unreliable for use in economic analysis.

To create a meaningful economic analysis, we must determine the *true* differences between alternatives, and that might require some adjustment of cost-accounting data. The following example illustrates this situation.

#### **EXAMPLE 1-1**

The cost-accounting records of a large company show the average monthly costs for the three-person printing department. The wages and benefits of the department make up the first category of *direct* labour. The company's indirect or overhead costs—such as heat, electricity, and employee insurance—must be distributed to its various departments in *some* manner and, like many other firms, this one uses *floor space* as the basis for its allocations.

Direct labour (including employee benefits)	\$6,000
Materials and supplies consumed	7,000
Allocated overhead costs:	
200 m <sup>2</sup> of floor area at \$25/m <sup>2</sup>	5,000
	\$18,000

The printing department charges the other departments for its services to recover its \$18,000 monthly cost. For example, the charge to run 1,000 copies of an announcement is

Direct labour	\$7.60
Materials and supplies	9.80
Overhead costs	9.05
Cost to other departments	\$26.45

The shipping department checks with a commercial printer that would print the same 1,000 copies for \$22.95. Although the shipping department needs only about 30,000 copies printed a month, its foreman decides to stop using the printing department and have the work done by the outside printer. The inhouse printing department objects to this. As a result, the general manager has asked you to study the situation and recommend what should be done.

#### **SOLUTION**

Much of the printing department's output reveals the company's costs, prices, and other financial information. The company president considers the printing department necessary in order to prevent disclosing such information to people outside the company.

A review of the cost-accounting charges reveals nothing unusual. The charges made by the printing department cover direct labour, materials and supplies, and overhead. The allocation of indirect costs is a customary procedure in cost-accounting systems, but it may be misleading for decision making, as the following discussion indicates.

	<b>Printing Department</b>		Outside Printer	
	1,000 Copies	30,000 Copies	1,000 Copies	30,000 Copies
Direct labour	\$7.60	\$228.00	_	_
Materials and supplies	9.80	294.00	\$22.95	\$688.50
Overhead costs	$\frac{9.05}{$26.45}$	$\frac{271.50}{\$793.50}$	<del></del> \$22.95	<del></del>

The shipping department would reduce its cost from \$793 to \$688 by using the outside printer. In that case, by how much would the printing department's costs decline? We will examine each of the cost components:

- 1. *Direct Labour*. If the printing department had been working overtime, then the overtime could be reduced or eliminated. But, assuming there was no overtime, how much would the saving be? It seems unlikely that a printer could be fired or even put on less than a 40-hour work week. Thus, although there might be a \$228 saving, it is much more likely that there will be no reduction in direct labour.
- 2. *Materials and Supplies.* There would be a \$294 saving in materials and supplies.
- 3. Allocated Overhead Costs. There will be no reduction in the printing department's monthly \$5,000 overhead, because there will be no reduction in department floor space. (Though there may be a slight reduction in the firm's power costs if the printing department does less work.) Furthermore, the firm will incur additional expenses in purchasing and accounting to deal with the outside firm.

The firm will save \$294 in materials and supplies and may or may not save \$228 in direct labour if the printing department no longer does the shipping department's work. The maximum saving would be \$294 + 228 = \$522. But if the shipping department obtains its printing from the outside printer, the firm must pay \$688.50 a month. The saving from not doing the shipping department work in the printing department would not exceed \$522, and it would probably be only \$294. The result would be a net increase in cost to the firm. For this reason, the shipping department should be discouraged from sending its printing to the outside printer.

Gathering cost data presents other difficulties. One way to look at the financial consequences—costs and benefits—of various alternatives is as follows:

- *Market Consequences*. These consequences have an established price in the marketplace. We can quickly determine raw material prices, machinery costs, labour costs, and so forth.
- Extra-Market Consequences. There are other items that are not directly priced in the marketplace. But by indirect means, a price may be assigned to these items. (Economists call these prices **shadow prices**.) Examples might be the cost of an employee's injury or the value to employees of going from a five-day to a four-day 40-hour week.
- Intangible Consequences. Numerical economic analysis never fully describes the differences between alternatives. We are tempted to leave out consequences that do not have a significant effect on the analysis itself or on the conversion of the final decision into its cash equivalent. How does one evaluate the potential loss of workers' jobs because of automation? What is the value of landscaping around a factory? These and similar factors may be left out of the numerical calculations, but they should be considered in conjunction with the numerical results in reaching a decision.

#### 4. Identify Feasible Alternatives

It should be kept in mind that unless the best alternative is considered, the result will always be less than ideal. Two types of alternatives are sometimes ignored. First, in many situations a "do-nothing" alternative is feasible. Second, there are often feasible (but unglamorous) alternatives, such as "Patch it up and keep it running for another year before replacing it."

There is no foolproof way to ensure that the best alternative is among the ones being considered. You should ensure that all conventional alternatives have been listed and then strive to suggest innovative solutions. Sometimes it can be useful for a group to consider alternatives in an innovative atmosphere, that is, by *brainstorming*. Even impractical alternatives may lead to a better possibility.

#### 5. Select the Criteria for Determining the Best Alternative

To choose the best alternative, we must define what we mean by *best*. There must be at least one *criterion*, or a set of criteria, to evaluate which alternative is best—or, in some unfortunate cases, the least bad. Here are several possible criteria:

- Create minimal disturbance to the environment.
- Improve the distribution of wealth among people.
- Minimize the expenditure of money.
- Ensure that the benefits to those who gain from the decision are greater than the losses of those who are harmed by the decision.<sup>1</sup>
- Minimize the time needed to accomplish the goal or objective.
- Minimize unemployment.
- Maximize profit.

Selecting the criteria by which to choose the best alternative will not be easy if different groups support different criteria and hence desire different alternatives. Or the criteria may conflict. For example, minimizing unemployment may increase the expenditure of money. Minimizing environmental disturbance may increase the time to complete the project. Disagreements between management and labour may reflect disagreements over the criteria for choosing the best alternative.

The last criterion—maximize profit—is the one normally selected in engineering decision making.

#### 6. Construct a Model

At some point in the decision making, the various elements must be brought together. The *objective*, *relevant data*, *feasible alternatives*, and *selection standards* must be merged.

Constructing the relationships between the decision-making elements is frequently called **model building**. To an engineer, a model may be a physical representation of the real thing or a set of equations describing the desired relationships. In economic decision making, the model is usually mathematical.

It is helpful to model only that part of the system important to the problem at hand. Thus the mathematical model of the student capacity of a classroom might be

Capacity = 
$$\frac{lw}{k}$$

<sup>&</sup>lt;sup>1</sup> This is the Kaldor criterion.

where l = length of classroom, in metres

w = width of classroom, in metres k = classroom arrangement factor

The equation for student capacity of a classroom is a very simple model, yet it may be adequate for the problem being solved.

#### 7. Predict the Outcomes for Each Alternative

A model and the data are used to predict the outcomes for each feasible alternative. As suggested earlier, each alternative might produce a variety of outcomes. Choosing a motorcycle rather than a bicycle, for example, may make the fuel supplier happy, the neighbours unhappy, the environment more polluted, and the owner's savings account smaller. But, to avoid unnecessary complications, we assume that decisions are based on a single criterion for measuring the relative attractiveness of the various alternatives. If necessary, one could devise a single composite criterion that is the weighted average of several different criteria.

To choose the best alternative, the outcomes for each alternative must be stated in a *comparable* way. Usually the consequences of each alternative are stated in terms of money. The consequences can be categorized as follows:

- Market consequences—where established market prices are available
- Extra-market consequences—no direct market prices, so priced indirectly
- Intangible consequences—valued by judgment, not monetary prices

In the first problems that we will examine, the costs and benefits occur over a short period and can be considered to occur at the same time. More commonly, the various costs and benefits take place over a longer period. We will represent these as a *cash flow diagram* to show the timing of the various costs and benefits.

For these longer-term problems, a common mistake is to assume that the current situation will be unchanged if the do-nothing alternative is chosen. For example, current profits may shrink or vanish as a result of the actions of competitors and the expectations of customers. As another example, traffic jams normally increase over the years as the number of vehicles increases.

#### 8. Choose the Best Alternative

Earlier we said that choosing the best alternative may be simply a matter of determining which best meets the selection criterion. But the solutions to most problems in economics have market consequences, extra-market consequences, and intangible consequences. Since the intangible consequences of possible alternatives are left out of the numerical calculations, they should be introduced into the decision making at this point. The right choice is the one that best meets the criteria after we have considered both the numerical and intangible consequences.

During the decision making, certain feasible alternatives are eliminated because they are *dominated* by other, better alternatives. For example, shopping for a computer online may allow you to buy a custom-configured computer for less money than a stock computer in a local store. Buying at the local store is feasible, but dominated.

Having examined the structure of the decision-making process, we can ask: When is the decision made, and who makes it? If one person performs *all* the steps in decision making, then she is the decision maker. *When* she makes the decision is less clear. The selection of the feasible alternatives may be the crucial step, with the rest of the analysis a methodical process leading to the inevitable decision.

#### 9. Audit the Results

An audit of the results is a comparison of what happened against the predictions. Do the results of a decision analysis reasonably agree with its projections? If a new machine tool was purchased to save labour and improve quality, did it? If the savings are not being obtained, what was overlooked? The audit may help ensure that the projected operating advantages are ultimately obtained. On the other hand, the economic analysis projections may have been unduly optimistic. We want to know this, too, so that the mistakes that led to the inaccurate projection are not repeated. An effective way to promote *realistic* economic analysis calculations is for all people involved to know that there *will* be an audit of the results.

#### **Ethics**

You must be mindful of the ethical dimensions of engineering economic analysis and of your engineering and personal decisions. This text can only introduce the topic, which we hope you will explore in greater depth.

Ethics can be described as distinguishing right and wrong when making decisions. This may include establishing systems of beliefs and moral obligations, defining values and fairness, and determining duties and guidelines for conduct. Ethical decision making requires an understanding of the context of the problem, the possible choices, and the outcomes of each choice.

#### **Ethical Dimensions in Engineering Decision Making**

Ethical issues can arise at every stage of the decision-making process. Ethics is such an important part of professional and business decisions that ethical codes or standards of conduct exist for professional engineering societies and small and large organizations, to guide individual and corporate behaviour. Written professional codes are common in the engineering profession, where they serve as a reference for new engineers and a basis for legal action against engineers who violate the code.

In Canada, provincial and territorial associations of professional engineers are responsible for the regulation of the practice of engineering. The federation of these organizations is called Engineers Canada. This is the Engineers Canada code of ethics:

Registrants shall conduct themselves with integrity and in an honourable and ethical manner. Registrants shall uphold the values of truth, honesty, and trustworthiness and safeguard human life and welfare and the environment. In keeping with these basic tenets, registrants shall

- 1. hold paramount the safety, health, and welfare of the public and the protection of the environment, and promote health and safety within the workplace;
- 2. offer services, advise on or undertake engineering assignments only in areas of their competence, and practise in a careful and diligent manner and in compliance with applicable legislation;
- 3. act as faithful agents of their clients or employers, maintain confidentiality, and avoid conflicts of interest, but, where such conflict arises, fully disclose the circumstances without delay to the employer or client;
- 4. keep themselves informed in order to maintain their competence, and advance their knowledge in the field within which they practise;

- 5. conduct themselves with integrity, equity, fairness, courtesy, and good faith toward clients, colleagues, and others, give credit where it is due, and accept, as well as give, honest and fair professional criticism;
- 6. present clearly to employers and clients the possible consequences if engineering decisions or judgments are overruled or disregarded;
- 7. report to their association or other appropriate agencies any illegal or unethical engineering decisions or practices by engineers or others;
- 8. be aware of and ensure that clients and employers are made aware of societal and environmental consequences of actions or projects, and endeavour to interpret engineering issues to the public in an objective and truthful manner;
- 9. treat equitably and promote the equitable treatment of people and in accordance with human rights legislation;
- 10. uphold and enhance the honour and dignity of the profession. (CCPE 2012)

In addition, each provincial and territorial association has a code of ethics for its members. Most engineering organizations have similar written standards. For all engineers, difficulties arise when their actions are contrary to these written or internal codes.

#### The Environment We Live In

The decision maker must ask who incurs the costs for the project and who receives the benefits. Ethical issues can be particularly difficult because there are often stakeholders with opposing viewpoints, and some of the data may be uncertain and hard to quantify.

The following are examples of difficult choices:

- Protecting the habitat of an endangered species versus flood-control projects that protect people, animals, and structures
- Meeting the need for electrical power when every means of generation causes some environmental damage:
  - Hydroelectric—loss of land and habitat to reservoir
  - Coal—danger to workers from underground mining, damage to habitat from open-pit mining, air pollution from burning the coal, and an increased rate of global warming because of the release of carbon dioxide to the atmosphere.
  - Nuclear—disposal of radioactive waste
  - Fuel oil—air pollution and economic dependence on foreign sources
  - Wind—visual pollution of wind farms, killing of birds and bats by whirling blades
- Determining standards for pollutants: Is 1 part per million acceptable, or is 1 part per billion necessary?

#### Safety and Cost

Some of the most common and most difficult ethical dilemmas involve trade-offs between safety and cost. If a product is "too safe," it will be too expensive and it will not be used. And sometimes the cost is borne by one party and the risk by another.

- Should the oil platform be designed for the 100-year, 500-year, or 1,000-year hurricane?
- Should the auto manufacturer add run-flat tires, stability control, side-cushion airbags, and rear-seat airbags to every car?
- Are stainless steel valves required, or is it economically better to use less-corrosionresistant valves and replace them more often?

#### Emerging Issues and "Solutions"

Breaches of the law by the corporate leaders of Enron, Tyco, and other firms led to attempts by governments to prevent, limit, and expose financial wrongdoing within corporations. One part of the American response has been the *Sarbanes-Oxley Act* of 2002, which imposed requirements on executives and auditing firms, and penalties for violations.

Globalization is another area of increasing importance for ethical considerations. One reason is that different countries and regions have different ethical expectations. A second reason is that jobs may be moved to another country because of differences in cost, productivity, environmental standards, and so on. What may from a Canadian perspective be a sweatshop may from the perspective of a less developed nation be an opportunity to support many families.

#### Importance of Ethics In Engineering and Engineering Economy

Frequently, though engineers and firms try to act ethically, mistakes are made—the data were wrong, the design was changed, or the operating environment was different than expected. But in other cases, such as the *Challenger* launch decision, a choice was made to place expediency above ethics. Under such circumstances, it is the engineers' duty to speak out, within and if necessary beyond their company, to protect the safety of the public.

Often, recent engineering graduates are asked, "What is the most important thing you want from your supervisor?" The most common response is mentoring and opportunities to learn and progress. When employees with 5, 15, 25, or more years of experience are asked the same question, the most common response is *integrity*. This is what your subordinates, peers, and superiors will expect and value the most from you. Integrity is the foundation for long-term career success.

# **Engineering Decision Making for Current Costs**

Some of the easiest kinds of engineering decisions deal with problems of alternative *designs*, *methods*, or *materials*. If the results of the decision appear in a very short time, one can quickly add up the costs and benefits for each alternative. Then, by using suitable economic standards, one can identify the best alternative. Three examples illustrate these situations.

#### **EXAMPLE 1-2**

A concrete aggregate mix must contain at least 31% sand by volume for proper batching. One source of material, which has 25% sand and 75% coarse aggregate, sells for \$3.00 per cubic metre (m³). Another source, which has 40% sand and 60% coarse aggregate, sells for \$4.40/m³. Determine the least cost per cubic metre of blended aggregates.

#### **SOLUTION**

The least cost of blended aggregates will result from maximum use of the lower-cost material. The higher-cost material will be used to increase the proportion of sand up to the minimum level (31%) specified.

Let x = Portion of blended aggregates from \$3.00/m<sup>3</sup> source

1 - x = Portion of blended aggregates from \$4.40/m<sup>3</sup> source

Sand Balance

$$x(0.25) + (1+x)(0.40) = 0.31$$
$$0.25x + 0.40 - 0.40x = 0.31$$
$$x = \frac{0.31 - 0.40}{0.25 - 0.40} = \frac{-0.09}{-0.15}$$
$$= 0.60$$

Thus the blended aggregates will contain

60% of \$3.00/m³ material 40% of \$4.40/m³ material

The least cost per cubic metre of blended aggregates is

$$0.60(\$3.00) + 0.40(\$4.40) = 1.80 + 1.76$$
  
=  $\$3.56/\text{m}^3$ 

#### **EXAMPLE 1-3**

A machine part is manufactured at a unit cost of 40¢ for material and 15¢ for direct labour. An investment of \$500,000 in tooling is required. The order calls for 3 million pieces. Halfway through the order, a new method of manufacture can be put into effect that will reduce the unit costs to 34¢ for material and 10¢ for direct labour—but it will require \$100,000 for additional tooling. This tooling will not be useful for future orders. Other costs are allocated at 2.5 times the direct labour cost. What, if anything, should be done?

#### **SOLUTION**

Since there is only one way to handle the first 1.5 million pieces, our problem concerns only the second half of the order.

Alternative A: Continue with Present Method

 Material cost
  $1,500,000 \text{ pieces} \times 0.40 =$  \$600,000

 Direct labour cost
  $1,500,000 \text{ pieces} \times 0.15 =$  225,000

 Other costs
  $2.50 \times \text{direct labour cost} =$  562,500

 Cost for remaining 1,500,000 pieces
 \$1,387,500

continued

#### Alternative B: Change the Manufacturing Method

Additional tooling cost		\$100,000
Material cost	$1,500,000 \text{ pieces} \times 0.34 =$	510,000
Direct labour cost	$1,500,000 \text{ pieces} \times 0.10 =$	150,000
Other costs	$2.50 \times \text{direct labour cost} =$	375,000
Cost for remaining	\$1,135,000	

Before making a final decision, you should closely examine the *other costs* to see that they do, in fact, vary as the *direct labour cost*. If they do, the decision would be to change the manufacturing method.

#### **EXAMPLE 1-4**

In the design of a cold-storage warehouse, the specifications call for a maximum heat transfer through the warehouse walls of 8 watts per square metre of wall when there is a 30°C temperature difference between the inside surface and the outside surface of the insulation. The two insulation materials being considered are listed below:

		Conductivity
Insulation Material	Cost per Cubic Metre	(W/m·°C)
Rock wool	\$12.50	0.039
Foamed insulation	14.00	0.03

The basic equation for heat conduction through a wall is

$$Q = \frac{K(\Delta T)}{L}$$

where  $Q = \text{heat transfer, in W/m}^2 \text{ of wall}$ 

 $K = \text{conductivity in W/m} \cdot ^{\circ}\text{C}$ 

 $\Delta T$  = difference in temperature between the two surfaces, in °C

L = thickness of insulating material, in metres

Which insulation material should be selected?

#### **SOLUTION**

Two steps are needed to solve the problem. First, the required thickness of each of the alternative materials must be calculated. Then, since the problem is one of providing a fixed output (heat transfer through the wall is limited to a fixed maximum amount), the criterion is to minimize the input (cost).

#### **Required Insulation Thickness**

Rock wool 
$$8 = \frac{0.039(30)}{L} \qquad L = 0.15 \text{ m}$$
 Foamed insulation 
$$8 = \frac{0.03(30)}{L} \qquad L = 0.11 \text{ m}$$

#### Cost of Insulation per Square Metre of Wall

Unit cost =  $Cost/m^3 \times Insulation thickness$ , in metres

Rock wool Unit cost =  $$12.50 \times 0.15 \text{ m} = $1.87/\text{m}^2$$ Foamed insulation Unit cost =  $$14.00 \times 0.11 \text{ m} = $1.54/\text{m}^2$$ 

The foamed insulation is the cheaper alternative. However, there are several intangible constraints that must be considered. Often, insulation material comes in a small number of standard thicknesses. We note that 0.15 m is very close to six inches, so we can probably obtain rock wool in that thickness or a multiple of it. How thick is the available wall space? What is the cost of more insulation versus the cost of cooling the warehouse over its life?

# **Engineering Costs**

Evaluating a set of alternatives requires that many costs be analyzed. In this section we describe several concepts for classifying and understanding these costs.

#### Fixed, Variable, Marginal, and Average Costs



**Fixed costs** are constant or unchanging regardless of the level of output or activity. In contrast, **variable costs** depend on the level of output or activity. A **marginal cost** is the variable cost for one more unit, while the **average cost** is the total cost divided by the number of units.

For example, many universities charge full-time students a fixed cost for 12 to 18 credit hours and a cost per credit hour for each credit hour over 18. Thus, for full-time students who are taking an overload (more than 18 hours), there is a *variable* cost that depends on the level of activity.

This example can also be used to distinguish between *marginal* and *average* costs. A marginal cost is the cost of one more unit. This will depend on how many credit hours the student is taking. If a student is currently enrolled for 12 to 17 hours, one additional hour is free. The marginal cost of an additional credit hour is \$0. However, if the student is taking 18 or more hours, the marginal cost is the variable cost of one more hour.

To illustrate average costs, the fixed and variable costs need to be specified. Suppose the cost of 12 to 18 hours is \$1,800 per term and overload credits are \$120 an hour. If a student takes 12 hours, the *average* cost is \$1,800/12 = \$150 per credit hour. If the student were to take 18 hours, the *average* cost would decrease to \$1,800/18 = \$100 per credit hour. If the student takes 21 hours, the *average* cost is \$102.86 per credit hour [ $$1,800 + (3 \times $120)/21$ ]. Average cost is thus calculated by dividing the total cost for all units by the total number of units. Decision makers use average cost to attain an overall cost picture of the investment on a per-unit basis.

#### **EXAMPLE 1-5**

The Federation of Student Societies of Engineering (FeSSE) wants to offer a one-day training course to help students find jobs and to raise funds for the Federation. The organizing committee is sure that they can find alumni, local business people, and faculty to provide the training at no charge. Thus the main costs will be for space, meals, handouts, and advertising.

continued

The organizers have classified the costs for room rental, room set-up, and advertising as fixed costs. They also have included the meals for the speakers as a fixed cost. Their total of \$225 is pegged to a room that will hold 40 people. If demand is greater than 40, the fixed costs will also increase.

The variable costs for food and bound handouts will be \$20 per student. The organizing committee believes that \$35 is about the right price to match value to students with their budgets. Since FeSSE has not offered training courses before, they are unsure how many students will reserve seats.

Develop equations for FeSSE's total cost and total revenue, and determine the number of registrations needed for revenue to equal cost.

#### **SOLUTION**

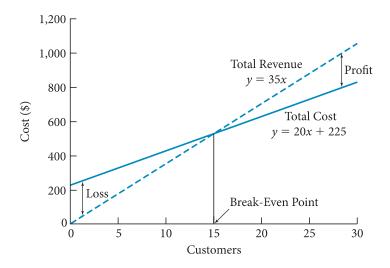
Let *x* be the number of students who sign up. Then,

$$Total cost = $225 + $20x$$
$$Total revenue = $35x$$

To find the number of student registrants needed, we equate these quantities and solve.

Total cost = Total revenue  

$$$225 + $20x = $35x$$
  
 $$225 = ($35 - $20)x$   
 $x = 225/15 = 15$  students



**FIGURE 1-2** Profit-loss break-even chart for Example 1-5.

From Example 1-5 we see how it is possible to calculate total fixed and total variable costs. Furthermore, these values can be combined into a single total-cost equation as follows:

$$Total cost = Total fixed cost + Total variable cost$$
 (1-1)

Example 1-5 developed *total cost* and *total revenue* equations to describe the training course proposal. These equations can be used to create what is called a *profit-loss break-even chart* (see Figure 1-2). Both the *costs* and *revenues* associated with various levels of

output (activity) are placed on the same set of x-y axes. This makes it possible to illustrate a *break-even point* and regions of *profit* and *loss* for some business activity. These terms can be defined as follows:

*Break-even point*: The level of activity at which the total cost of providing the product, good, or service is *equal to* the revenue (or savings) generated.

*Profit region*: Values of the variable *x* greater than the break-even point, where total revenue is greater than total costs.

*Loss region*: Values of the variable *x* less than the break-even point, where total cost is greater than total revenue.

Notice in Figure 1-2 that the break-even point for the number of people in the training course is 15 people. For more than 15 people, FeSSE will make a profit. If fewer than 15 sign up, there will be a loss.

The fixed costs of our simple model are only fixed over a certain range of values for x. In Example 1-5, that range was 1 to 40 students. If zero students sign up, the course could be cancelled and many of the fixed costs would not be incurred. Some costs, such as advertising, might already have been incurred, and there might be cancellation fees. If more than 40 students sign up, there would be greater costs for larger rooms or multiple sessions.

When modelling a specific situation, we often use linear variable costs and revenues. However, sometimes the relationship may be non-linear. For example, employees are often paid at 150% of their hourly rate for overtime hours, so that production levels requiring overtime have higher variable costs. Total cost in Figure 1-3 is a fixed cost of \$3,000 plus a variable cost of \$200 per unit for straight-time production of up to 10 units and \$300 per unit for overtime production of up to five more units.

Figure 1-3 can also be used to illustrate marginal and average costs. At a volume of five units the marginal cost is \$200 per unit, whereas at a volume of 12 units the marginal cost is \$300 per unit. At five units the average cost is \$800 per unit, or  $(3,000 + 200 \times 5)/5$ . At 12 units the average cost is \$467 per unit, or  $(3,000 + 200 \times 10 + 300 \times 2)/12$ .

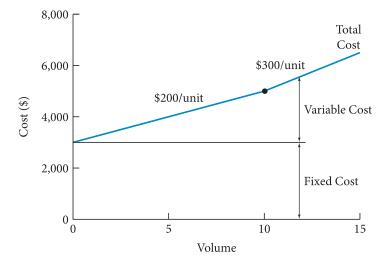


FIGURE 1-3 Non-linear variable costs.

#### **Sunk Costs**



A **sunk cost** is money already spent as a result of a *past* decision. Sunk costs should be disregarded in our engineering economic analysis because current decisions